Entangling the ro-vibrational modes of a macroscopic mirror using radiation pressure

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We consider the dynamics of a vibrating and rotating end-mirror of an optical Fabry-Pérot cavity that can sustain Laguerre-Gaussian modes. We demonstrate theoretically that since the intracavity field carries linear as well as angular momentum, radiation pressure can create bipartite entanglement between a vibrational and a rotational mode of the mirror. Further we show that the ratio of vibrational and rotational couplings with the radiation field can easily be adjusted experimentally, which makes the generation and detection of entanglement robust to uncertainties in the cavity manufacture. This constitutes the first proposal to demonstrate entanglement between two qualitatively different degrees of freedom of the same macroscopic object.

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No known law of physics prevents the application of quantum mechanics to macroscopic bodies. Characteristic traits of quantum mechanics such as entanglement [1] can then in principle be displayed even by large objects. The demonstration of quantum entanglement in macroscopic objects would clearly be interesting from the point of view of fundamental considerations, such as the exploration of the quantum-classical boundary [2]. It is also expected to have important applied consequences since entanglement is a crucial resource for information processing enabling quantum communication, computation, and measurement, see Ref. [3] and references therein.

Various mechanisms have been proposed for generating entanglement between different degrees of freedom of macroscopic objects. The flexural modes of nanomechanical electrodes can be entangled by the ions they trap, via Coulomb interactions [4]. Entanglement can be generated between the vibrations of an array of gold beams fabricated on a semiconductor membrane using electric voltages [5]. The motion of a nanomechanical oscillator carrying a ferromagnetic domain can become entangled with the collective spin of a mesoscopic Bose Einstein condensate due to their magnetic coupling [6]. Radiation pressure can entangle two vibrating nanofabricated mirrors that may belong to an optical cavity [7, 8, 9, 10, 11] or not [12]. In addition a single cavity mode can also entangle multiple vibrational modes of the same mirror [8].

In this Letter we discuss instead how radiation pressure can entangle two *qualitatively* different motional degrees of freedom of the same classical object. Specifically we show that the optomechanical coupling produced by a Laguerre-Gaussian intracavity field can lead to bipartite entanglement between the modes of rotation and vibration of a moving mirror. Further, we demonstrate that the ratio of the coupling of the optical field to the vibration and rotation modes can be adjusted experimentally, resulting in the robust generation of entanglement against uncertainties in the mass, radius and mechanical frequencies of the mirror. This is in contrast to previous proposals in which entanglement relies crucially on the precise balance of mirror parameters, a situation that is difficult to attain in practice [7, 8, 9, 10, 11, 13].

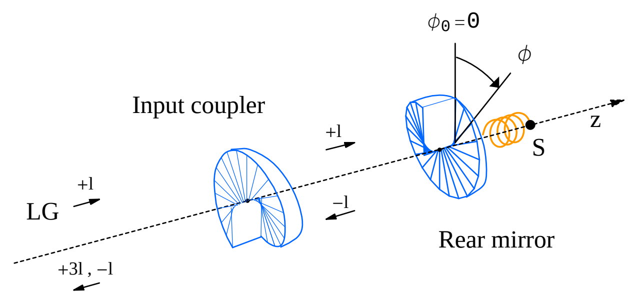

FIG. 1: (Color online). The arrangement proposed in this work for entangling the vibrational and rotational modes of a mirror. A Laguerre-Gaussian beam is incident on the resonant cavity formed by two spiral phase elements, the transmissive but fixed input coupler and the perfectly reflective moving rear mirror. The rear mirror is mounted on a helical spring S which provides a vibrating restoring force along the z axis as well as torsion opposite the direction ϕ. The deflection of the rear mirror from its angular equilibrium position ($\phi_0 = 0$) is indicated by the angle ϕ; the z deflection of the mirror has not been shown for clarity. The charge on the Laguerre-Gaussian beams at various points has been indicated.

The configuration that we consider consists of an optical cavity formed by two spiral phase elements, see Fig. 1. Spiral phase elements can be reflective or transmissive and are used to change the angular momentum or 'optical charge' of laser beams [14]. The input coupler transmits light weakly, and without changing its charge. A beam reflected from it however gains a charge $2l$. The rear mirror on the other hand reflects light perfectly, removing at the same time a charge $2l$ from it. Designed in this manner the cavity can provide mode build-up to an incident Laguerre-Gaussian field of charge $+l$; a detailed discussion of the cavity conditions has been given in Ref. [14].

The input coupler is supported rigidly, but the rear mirror is mounted in such a way that it can vibrate as well as rotate. One way to accomplish this may be by mounting the mirror on a helical spring [15]. A number of experiments have demonstrated that the linear vibrations of the mirror can be cooled by a Gaussian cavity mode [16]; in a recent proposal we have shown that if a Laguerre-Gaussian mode is used instead rotational cooling can also be achieved [14]. In the current design we combine both effects – since each Laguerre-Gaussian photon carries linear as well as angular momentum, the same cavity mode can affect both the vibration as well as the rotation of the mirror.

The coupling of radiation to the mirror motion can be derived by using the fact that both the linear and the angular momentum of the incident $+l$ Laguerre-Gaussian beam are reversed by the rear mirror. The torque per intracavity photon is the rate of change of angular momentum $2l\hbar/(2L/c)$, and similarly for the force, with rate of change of momentum $2\hbar k/(2L/c)$. Here k is the wave vector of the light field, L is the length of the cavity and c is the velocity of light [14].

Using these arguments we can model the physical system described above and shown in Fig. 1 by the Hamiltonian

$$\begin{aligned} H &= \hbar\omega_c a^\dagger a + \frac{\hbar\omega_z}{2}(p_z^2 + z^2) + \frac{\hbar\omega_\phi}{2}(L_z^2 + \phi^2) \\ &- \hbar g_z a^\dagger a z + \hbar g_\phi a^\dagger a \phi. \end{aligned} \tag{1}$$

The first term in this Hamiltonian describes the electromagnetic energy of the cavity mode, the next two terms the vibrational and rotational energies of the moving mirror and the last two terms the effects of radiation force and torque on the rear mirror respectively.

In Eq. (1) a and $a^\dagger$ are the bosonic annihilation and creation operators for the cavity mode. z and p_z are the dimensionless position and momentum of the mirror scaled to the characteristic length $(\hbar/M\omega_z)^{1/2}$ and momentum $(\hbar M\omega_z)^{1/2}$, M being the mass of the mirror and ω_z its frequency of linear vibration. Similarly ϕ and L_z are the dimensionless mirror angular displacement and momentum scaled to $(\hbar/I\omega_\phi)^{1/2}$ and $(\hbar I\omega_\phi)^{1/2}$ respectively, $I = MR^2/2$ being the moment of inertia, R the mirror radius, and ω_ϕ the frequency of angular vibration. The commutation relations of the dynamical variables are given by $[a, a^\dagger] = 1$, $[z, p_z] = i$, and $[\phi, L_z] = i$ respectively. The frequency $\omega_c = n\pi c/L$ is the cavity mode frequency, where $L = n\lambda/2$. Finally, the *opto-vibrational* and *opto-rotational* coupling constants are given by

$$g_z = \frac{\omega_c}{L}\sqrt{\frac{\hbar}{M\omega_z}}, \quad g_\phi = \frac{cl}{L}\sqrt{\frac{\hbar}{I\omega_\phi}}, \tag{2}$$

respectively.

We consider the Heisenberg equations of motion for the dynamical variables of the Hamiltonian (1), adding damping and noise to arrive at the nonlinear quantum Langevin equations for the system [17]:

$$\begin{aligned} \dot{a} &= -i(\delta - g_z z + g_\phi \phi)a - \frac{\gamma}{2}a + \sqrt{\gamma}a^{\rm in}, \\ \dot{z} &= \omega_z p_z, \\ \dot{p_z} &= -\omega_z z + g_z a^\dagger a - \gamma_z p_z + \epsilon_z^{\rm in}, \\ \dot{\phi} &= \omega_\phi L_z, \\ \dot{L_z} &= -\omega_\phi \phi - g_\phi a^\dagger a - \gamma_\phi L_z + \epsilon_\phi^{\rm in}. \end{aligned} \tag{3}$$

Here $\delta = \omega_c - \omega_L$ is the detuning of the laser frequency ω_L from the cavity resonance, γ is the damping rate of the cavity, γ_z and γ_ϕ the intrinsic damping rates of vibration and rotation respectively, and $a^{\rm in}$ is a noise operator describing the laser field incident on the cavity. The mean value $\langle a^{\rm in}(t)\rangle = a_s^{\rm in}$ describes the classical Laguerre-Gaussian field, and the delta-correlated fluctuations

$$\langle \delta a^{\rm in}(t)\delta a^{{\rm in},\dagger}(t')\rangle = \delta(t - t'), \tag{4}$$

describe the vacuum noise injected into the cavity mode by the driving field. The Brownian noise operator $\epsilon_z^{\rm in}$ accounts for the mechanical noise that couples into the mode of mirror vibration from the thermal environment. Its mean value is zero and its fluctuations are correlated at temperature T as [17]

$$\begin{aligned} &\langle \delta\epsilon_z^{\rm in}(t)\delta\epsilon_z^{\rm in}(t')\rangle = \\ &\frac{\gamma_z}{\omega_z}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\omega\left[1 + \coth\left(\frac{\hbar\omega}{2k_BT}\right)\right], \end{aligned} \tag{5}$$

where k_B is Boltzmann's constant. Similar relations hold for the rotational noise operator $\epsilon_\phi^{\rm in}$.

The steady-state values of the dynamical variables can be found from the equations

$$\begin{aligned} a_s &= \frac{\sqrt{\gamma}|a_s^{\rm in}|}{\left[(\frac{\gamma}{2})^2 + (\delta - a_s^2 G)^2\right]^{1/2}}, \\ z_s &= \frac{g_z a_s^2}{\omega_z}, \quad \phi_s = -\frac{g_\phi a_s^2}{\omega_\phi}, \\ p_{z,s} &= 0, \quad L_{z,s} = 0, \end{aligned} \tag{6}$$

where $G = g_z^2/\omega_z + g_\phi^2/\omega_\phi$, and the phase of the input field $a_s^{\rm in}$ has been chosen such that a_s is real. The field amplitude a_s is found by solving the first equation, which is nonlinear, and z_s and ϕ_s can then be determined. The solutions to Eq. (6) display bistability for high enough input power $P_{\rm in} = \hbar\omega_c|a_s^{\rm in}|^2$ [18]. In the rest of the paper we assume the use of electronic feedback, which allows us to set the net detuning $\Delta = \delta - a_s^2 G$ independently of radiation pressure and also to suppress bistability. Such

a procedure is carried out routinely in mirror cooling experiments [16].

To investigate the behavior of the system for small deviations away from its steady-state we expand every operator as the sum of a (steady state) mean value [Eq. (6)] and a small fluctuation, e.g. $a = a_s + \delta a$. Treating the other operators in Eq. (3) in a similar way and retaining only terms linear in the fluctuations yields

$$\begin{aligned}
\dot{\delta a} &= -(i\Delta + \frac{\gamma}{2})\delta a + i a_s (g_z \delta z - g_\phi \delta\phi) + \sqrt{\gamma}\delta a^{\mathrm{in}}, \\
\dot{\delta z} &= \omega_z \delta p_z, \\
\dot{\delta p_z} &= -\omega_z \delta z + g_z a_s(\delta a + \delta a^\dagger) - \gamma_z \delta p_z + \delta\epsilon_z^{\mathrm{in}}, \\
\dot{\delta\phi} &= \omega_\phi \delta L_z, \\
\dot{\delta L_z} &= -\omega_\phi \delta\phi - g_\phi a_s(\delta a + \delta a^\dagger) - \gamma_\phi \delta L_z + \delta\epsilon_\phi^{\mathrm{in}}.
\end{aligned} \tag{7}$$

We solve Eq. (7) in the frequency domain [19]. Combining the solutions of Eq. (7) with the relations in Eqs. (4) and (5) allows one to obtain the correlations between the quantum fluctuations of the dynamical variables, and hence the entanglement in the system [7]. This is because the fluctuations are continuous Gaussian variables fully determined by their first and second moments. Computable measures for bipartite entanglement between such variables exist and have been used previously to quantify optomechanical systems [19]. The calculation of entanglement in the frequency domain is also appropriate since the cavity dynamics are experimentally easier to probe spectrally than in the time domain [8].

More specifically, we consider the operators $\delta u = \delta z - \delta\phi$ and $\delta v = \delta p_z + \delta L_z$. We then construct the corresponding Hermitian operators $\mathcal{R}_{u,v}$, where $\mathcal{R}_u = [\delta u(\omega) + \delta u(-\omega)]/2$, for example. An entanglement measure $\mathcal{E}(\omega)$ can then be defined as [7]

$$\mathcal{E}(\omega) = \frac{\langle \mathcal{R}_u^2(\omega)\rangle \langle \mathcal{R}_v^2(\omega)\rangle}{|\langle[\mathcal{R}_z(\omega), \mathcal{R}_{p_z}(\omega)]\rangle|^2}. \tag{8}$$

Ro-vibrational entanglement exists at the system response frequency ω whenever $\mathcal{E}(\omega) < 1$.

Figure 2 plots $\mathcal{E}(\omega)$ as a function of the response frequency ω and temperature T for the experimentally accessible parameters detailed in the caption. A significant amount of entanglement is available at higher than cryogenic temperatures, a regime in which mirror cooling has been demonstrated [20], and at a usably large bandwidth.

We observe that the entanglement is always maximum at the arithmetic mean of the two mechanical frequencies, i.e. $\mathcal{E}_{\max} = \mathcal{E}\left((\omega_z + \omega_\phi)/2\right)$ [7, 8]. Further, we find the presence of the symmetry $g_z = g_\phi$ to be crucial to the generation of entanglement in our system. This is illustrated in Fig. 3 which plots the maximum entanglement $\mathcal{E}_{\max}$ as a function of the percent fractional imbalance in the couplings. It shows that entanglement vanishes rapidly even for small deviations away from equality. Further investigations indicate that for increasing coupling asymmetry the bandwidth of entanglement also vanishes faster with temperature. The requirement of a symmetric coupling in order to radiation-entangle mechanical modes has been noted previously in the case of mirror vibration [7, 8, 9, 10, 11, 13]. Very recently the effect of asymmetry has been precisely characterized for the case of general Gaussian continuous variables [21]. It

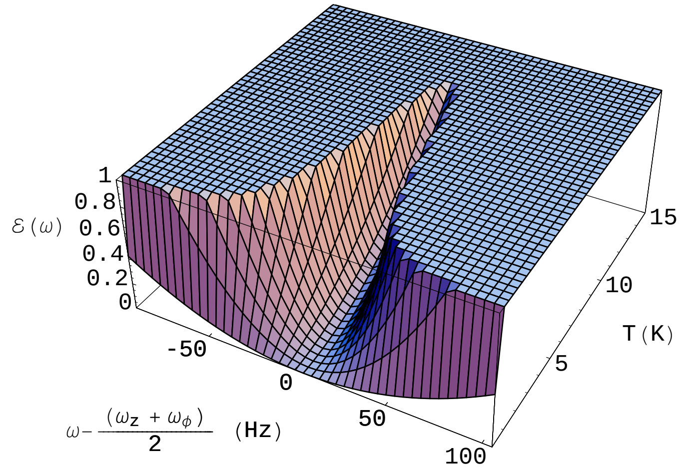

FIG. 2: (Color online). Radiation pressure-induced entanglement $\mathcal{E}(\omega)$ between a vibrational and a rotational mode of the moving mirror. The parameters are $M = 1\mu$g, $R = 15\mu$m, vibrational and rotational quality factors $Q_z = Q_\phi = 10^6$, $\omega_z \simeq \omega_\phi = 1$ MHz, $l = 82$, $L \simeq 4$ mm, cavity finesse $F = 2.5 \times 10^4$, $\lambda = 812.7$nm, $\Delta = \omega_\phi$, and $P_{in} = 1$mW.

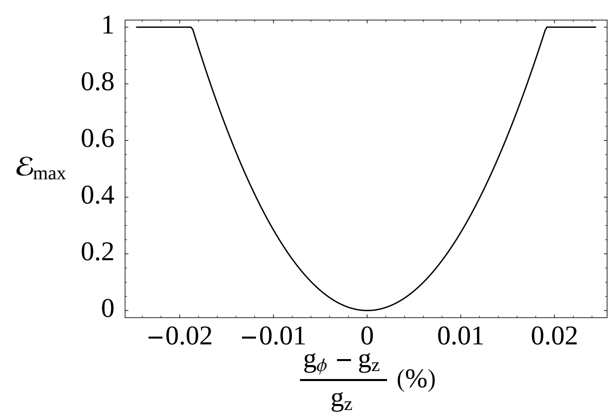

FIG. 3: Maximum entanglement $\mathcal{E}_{\max}$ at $T = 1$K between a vibration and a rotational mode of the rear mirror of Fig. 1 as a function of the percent fractional imbalance in the couplings with the radiation field. The entanglement decreases with increasing asymmetry in the couplings. In the figure for a 200 Hz difference between rotational and vibrational frequencies centered at 1 MHz, the coupling imbalance, about 0.02%, destroys the entanglement completely. The remaining parameters are the same as in Fig. 2.

has been shown analytically that higher the asymmetry, lower the entanglement.

It can be seen from the expression (2) for g_z that for the case of entanglement between two vibrational modes of the same mirror [8] the ratio of the couplings is determined by the frequencies of these modes. These may be difficult to control experimentally and are typically unequal. For our system the two mirror degrees of freedom couple differently to the field, with a ratio

$$\frac{g_z}{g_\phi} = \frac{2\pi}{l\lambda}\sqrt{\frac{I\omega_\phi}{M\omega_z}}. \tag{9}$$

In case the frequencies ω_z and ω_ϕ, mass M and moment of inertia I are all slightly different from their nominal values, it is possible to equalize the couplings by varying the radiation wavelength λ, simultaneously adjusting the cavity length L so as to stay on resonance.

We have used this procedure to arrive at the values presented in Fig. 2, where the imbalance in the couplings is assumed to be due to a frequency mismatch $\omega_\phi - \omega_z =$ 10 Hz. We found that the couplings could be equalized and the entanglement retained by tuning λ by $\sim$ 2nm and the cavity length by $\sim$ 100μm. Such adjustments are easily within reach of current experimental techniques. We finally note that the experimental measurement of the entanglement $\mathcal{E}(\omega)$ can be carried out using standard techniques such as homodyne measurements as described in Ref. [7]. This involves use of a secondary cavity with a third mirror beyond the rear mirror.

In conclusion we have demonstrated that a vibrational and a rotational mode of the same macroscopic mirror can be entangled quantum mechanically by radiation pressure from a Laguerre-Gaussian cavity mode. The entanglement can be made robust against imprecision in the cavity manufacture because the ratio of vibrational-to-rotational coupling with the radiation can be tuned experimentally.

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